

C O N T E N T S

PREFACE

1 THE ELEMENTS OF THE THEORY OF SETS

1.1	Introduction	1
1.2	The Concept of Set	3
1.3	Constants	4
1.4	Variables and Equality	7
1.5	Some Basic Notations and Definitions	10
1.6	Subsets; Equality of Sets; The Empty Set	12
1.7	The First Theorem	17
1.8	A (Very) Brief Section on Logic	18
1.9	The Algebra of Sets	25
1.10	Remarks on Notation and Other Matters	34
1.11	Some Special Sets	38
1.12	Ordered Pairs	43
1.13	Cartesian Products, Relations	46

1, by ALLYN AND BACON, INC.,
ue, Boston. All rights reserved. No
may be reproduced in any form, or
hout permission in writing from the

ess Catalog Card Number: 61-15038

August, 1961
December, 1967

1.14	Functions (or Mappings)	49
1.15	Equivalence Relations and Partitions	63
1.16	Mathematical Systems	72

2 THE NATURAL NUMBERS

2.1	The Definition of the Natural Numbers	74
2.2	The Ordering of the Natural Numbers	89
2.3	Counting	98
2.4	Finite Sets	101
2.5	Addition and Multiplication	106
2.6	The Relations between Order, Addition and Multiplication	112
2.7	The Principle of Finite Induction, Again	115
2.8	Sequences	117
2.9	Recursive Definitions	120

3 THE INTEGERS AND THE RATIONAL NUMBERS

3.1	Introduction	132
3.2	Definition and Properties of the Integers	133
3.3	Number-Theoretic Properties of the Integers: Generalized Operations	147
3.4	The Rational Numbers	157
3.5	The Arithmetic of the Rational Numbers	161
3.6	Conclusion: Integral Domains and Quotient Fields	173

4 THE REAL NUMBERS

4.1	The Mysterious $\sqrt{2}$	177
4.2	The Arithmetic of Sequences	180
4.3	Cantor Sequences	187
4.4	Null Sequences	194
4.5	The Real Numbers	199

	49
itions	63
	72
RS	
Numbers	74
Numbers	89
	98
	101
	106
Addition and Multiplication	112
on, Again	115
	117
	120
HE	
	132
Integers	133
of the Integers:	
	147
	157
Numbers	161
and Quotient Fields	173
	177
	180
	187
	194
	199

5 THE DEEPER STUDY OF THE REAL NUMBERS

5.1	Ordered Fields	214
5.2	Relations between Ordered Fields and R , the Field of Rational Numbers	221
5.3	The Completeness of the Real Numbers	227
5.4	Roots of Real Numbers	239
5.5	More Theorems on Ordered and Complete, Ordered Fields	244
5.6	The Isomorphism of Complete, Ordered Fields	249
5.7	The Complex Numbers	253